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HEAT CONDUCTION AND HEAT TRANSFER IN TECHNOLOGICAL PROCESSES

INFLUENCE OF THE HEAT EXCHANGE BETWEEN THE OUTER SURFACE OF A PLANE-PARALLEL STRIP-FOUNDATION TRIBOUNIT AND THE ENVIRONMENT ON ITS TEMPERATURE

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Asymptotics (for small and large times) for solving the thermal friction problem for a tribosystem consisting of a plane-parallel strip sliding at a constant velocity on the surface of a semi-infinite foundation have been found. The thermal strip-foundation contact is ideal, and on the outer surface of the strip convective heat exchange with the environment takes place. The corresponding thermal problem of friction under braking has also been solved. Numerical analysis has been performed for the materials of the frictional pair cermet layer–cast iron foundation.

Keywords: *heat generation by friction, temperature, heat exchange, drag braking.*

Introduction. The plane-parallel strip-foundation calculation scheme is used to determine the average temperature of friction pairs (e.g., lining-disc) operating under high-speed heating (large Peclét numbers). In this case, the major portion of heat generated on the contact surface propagates into the inside of the strip and the foundation perpendicular to this surface [1, 2]. Consequently, the corresponding thermal problems of friction can be formulated as one-dimensional boundary-value heat conduction problems of the parabolic type [3, 4].

A solution nas been found for the thermal problem of friction for a plane-parallel strip sliding with a constant velocity on a semi-infinite foundation for two variants of the strip: boundary conditions on the outer surface of the strip, maintaining on it in the process of sliding the initial (zero) temperature or zero intensity of the heat flow (heat insulation) [5]. This solution was generalized to the case of heat exchange on the outer surface of the strip with the environment by the Newton law in [6, 7]. The influence of the heat transfer through the contact surface (nonideal thermal contact) on the strip and foundation temperatures was investigated in [8].

One practical application of the obtained results is the solution on their basis of the corresponding thermal problems of friction under braking. For instance, with the use of the solution obtained in [5] we investigated the evolution and distribution of the temperature in a strip-foundation tribosystem in their relative equally retarded sliding [9].

This paper presents the asymptotics found by us for the solution obtained in [6, 7] for small and large times and shows the possibility of using it for calculating the temperature of a lining-disc tribosystem.

Formulation of the Problem. Let a plane-parallel strip of thickness d and a half-closed foundation (half-space) be compressed by a normal load of constant intensity p_0 applied to the outer surface of the strip and at infinity in the half-space (Fig. 1). At the initial instant of time $t = 0$ the strip begins to slide on the half-space surface with a constant velocity V_0 in the positive direction of the y axis. As a consequence of friction, on the contact surface heat is generated and the tribosystem is heated. The wear of the contacting surfaces and the heat transfer between the friction pair elements are neglected. Therefore, we assume the strip and foundation temperatures on the contact plane to be equal and the sum of intensities of the heat flows into the inside of each of the bodies to be equal to the specific friction power ($q_0 = fV_0p_0$). On the strip surface $z = d$, convective heat exchange with the environment occurs. All quantities and parameters pertaining to the strip and the foundation will further have subscripts s and f, respectively.

The thermal friction problem corresponding to the above assumption has the form

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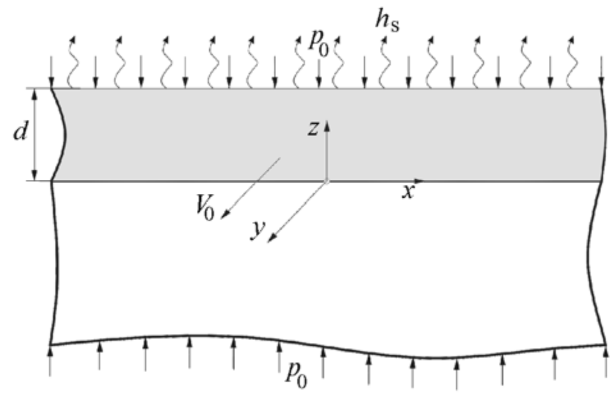


Fig. 1. Scheme of the strip-foundation contact.

$$\frac{\partial^2 T^*(\zeta,\tau)}{\partial \zeta^2} = \frac{\partial T^*(\zeta,\tau)}{\partial \tau}, \quad 0 < \zeta < 1, \quad \tau > 0, \tag{1}$$

$$\frac{\partial^2 T^*(\zeta,\tau)}{\partial \zeta^2} = \frac{1}{k^*}\frac{\partial T^*(\zeta,\tau)}{\partial \tau}, \quad -\infty < \zeta < 0, \quad \tau > 0, \tag{2}$$

$$T^*(0-,\tau) = T^*(0+,\tau), \quad \tau > 0, \tag{3}$$

$$K^* \left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta=0-} - \left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta=0+} = 1, \quad \tau > 0, \tag{4}$$

$$\left.\frac{\partial T^*}{\partial \zeta}\right|_{\zeta=1} + \mathrm{Bi_s}\, T^*(1,\tau) = 0, \quad \tau > 0, \tag{5}$$

$$T^*(\zeta,\tau) \to 0, \quad \zeta \to -\infty, \quad \tau > 0, \tag{6}$$

$$T^*(\zeta,0) = 0, \quad -\infty < \zeta \le 1, \tag{7}$$

where

$$\zeta = \frac{z}{d}, \quad \tau = \frac{k_s t}{d^2}, \quad K^* = \frac{K_f}{K_s}, \quad k^* = \frac{k_f}{k_s}, \quad \mathrm{Bi_s} = \frac{h_s d}{K_s}, \quad T^* = \frac{T}{T_0}, \quad T_0 = \frac{q_0 d}{K_s}. \tag{8}$$

Exact Solution of the Problem. The solution of the thermal friction problem (1)–(7) in the integral Laplace transform image space [10]

$$\overline{T}^*(\zeta,p) = \int_0^\infty T^*(\zeta,\tau)\exp(-p\tau)\,d\tau \tag{9}$$

has the form

$$\overline{T}^*(\zeta, p) = \frac{\Delta(\zeta, p)}{p\sqrt{p}\,\Delta(p)}, \tag{10}$$

where

$$\Delta(\zeta, p) = \begin{cases} \sqrt{p}\cosh(1-\zeta)\sqrt{p} + \mathrm{Bi_s}\sinh(1-\zeta)\sqrt{p}, & 0 \le \zeta \le 1, \\ (\sqrt{p}\cosh\sqrt{p} + \mathrm{Bi_s}\sinh\sqrt{p})\exp\left(\zeta\sqrt{\dfrac{p}{k^*}}\right), & -\infty < \zeta \le 0, \end{cases} \tag{11}$$

$$\Delta(p) = (\sqrt{p} + \varepsilon\,\mathrm{Bi_s})\sinh\sqrt{p} + (\varepsilon\sqrt{p} + \mathrm{Bi_s})\cosh\sqrt{p}; \tag{12}$$

$\varepsilon = K^*/\sqrt{k^*}$ is the thermal activity coefficient of the foundation with respect to the strip [11].

Going over, through the integration in the complex variable plane p, from transforms (10)–(12) to the originals, we find the dimensionless temperature fields in the strip and in the foundation [6, 7]

$$T^*(\zeta, \tau) = T_0^*(\zeta) - \frac{2}{\pi}\int_0^\infty F(x)\,G(\zeta, x)\exp(-x^2\tau)\,dx, \quad -\infty < \zeta \le 1, \quad \tau \ge 0, \tag{13}$$

where

$$T_0^*(\zeta) = \begin{cases} \dfrac{1 + (1-\zeta)\,\mathrm{Bi_s}}{\mathrm{Bi_s}}, & 0 \le \zeta \le 1, \\ \dfrac{1 + \mathrm{Bi_s}}{\mathrm{Bi_s}}, & -\infty < \zeta \le 0; \end{cases} \tag{14}$$

$$F(x) = \frac{\cos x + \mathrm{Bi_s}\,x^{-1}\sin x}{(\mathrm{Bi_s}\cos x - x\sin x)^2 + \varepsilon^2(\mathrm{Bi_s}\sin x + x\cos x)^2}; \tag{15}$$

$$G(\zeta, x) = \begin{cases} \varepsilon\,[\cos(1-\zeta)x + \mathrm{Bi_s}\,x^{-1}\sin(1-\zeta)x], & 0 \le \zeta \le 1, \\ \varepsilon\,(\mathrm{Bi_s}\,x^{-1}\sin x + \cos x)\cos\left(\dfrac{\zeta x}{\sqrt{k^*}}\right) - (\mathrm{Bi_s}\cos x - x\sin x)\,x^{-1}\sin\left(\dfrac{\zeta x}{\sqrt{k^*}}\right), & -\infty < \zeta \le 0. \end{cases} \tag{16}$$

We determine the maximum temperature by formulas (13)–(16) assuming in them $\zeta = 0$. In so doing, the function $G(\zeta, x)$ becomes equal to

$$G(0-, x) = G(0+, x) = \varepsilon\,(\cos x + \mathrm{Bi_s}\,x^{-1}\sin x). \tag{17}$$

From relations (13)–(15) and (17) it follows that the boundary condition (3) on the contact surface is fulfilled.

In view of designations of (8), we find the dimensionless intensities of the heat flows in the strip and the foundation

$$q^*(\zeta, \tau) \equiv \frac{q(z, t)}{q_0} = \begin{cases} -\dfrac{\partial T^*(\zeta, \tau)}{\partial \zeta}, & 0 \le \zeta \le 1, \quad \tau \ge 0, \\ K^*\dfrac{\partial T^*(\zeta, \tau)}{\partial \zeta}, & -\infty < \zeta \le 0, \quad \tau \ge 0, \end{cases} \tag{18}$$

from solutions (13)–(16) in the form

$$q^{*}(\zeta,\tau)=q_0^{*}-\frac{2\varepsilon}{\pi}\int\limits_0^{\infty}F(x)\,Q(\zeta,x)\exp(-x^2\tau)\,dx\,,\quad \tau\ge 0\,, \tag{19}$$

$$q_0^{*}=\begin{cases}1\,, & 0\le\zeta\le 1\,,\\ 0\,, & -\infty<\zeta\le 0\,,\end{cases} \tag{20}$$

$$Q(\zeta,x)=\begin{cases}\mathrm{Bi_s}\cos(1-\zeta)\,x-x\sin(1-\zeta)\,x\,, & 0\le\zeta\le 1\,,\\ (x\sin x-\mathrm{Bi_s}\cos x)\cos\left(\dfrac{\zeta x}{\sqrt{k^*}}\right)-\varepsilon\,(\mathrm{Bi_s}\sin x+x\cos x)\sin\left(\dfrac{\zeta x}{\sqrt{k^*}}\right), & -\infty<\zeta\le 0\,,\end{cases} \tag{21}$$

where the integrand $F(x)$ has the form of (15).

Assuming in formulas (21) $\zeta = 0$, we find $Q(0+, x) = -Q(0-, x) = \mathrm{Bi_s}\cos x - x\sin x$ and from relations (19), (20) $q^{*}(0+,\tau)+q^{*}(0-,\tau) = 1$ follows, which points to the fulfillment of the boundary condition (4).

Assuming in formulas (13)–(16) and (19)–(21) $\zeta = 1$, we find the dimensionless temperature and intensity of the heat flow on the outer surface of the strip:

$$T^{*}(1,\tau)=\frac{1}{\mathrm{Bi_s}}-\frac{2\varepsilon}{\pi}\int\limits_0^{\infty}F(x)\exp(-x^2\tau)\,dx\,,\quad q^{*}(1,\tau)=\mathrm{Bi_s}\,T^{*}(1,\tau)\,,\quad \tau\ge 0\,. \tag{22}$$

The second formula in (22) points to the fulfillment of the boundary condition (5).

Passing in formulas (14)–(16) and (21) to the limit $\mathrm{Bi_s}\to\infty$, we find

$$T_0^{*}(\zeta)=\begin{cases}1-\zeta\,, & 0\le\zeta\le 1\,,\\ 1\,, & -\infty<\zeta\le 0\,,\end{cases} \tag{23}$$

$$F(x)=\frac{x^{-1}\sin x}{\cos^2 x+\varepsilon^2\sin^2 x}\,, \tag{24}$$

$$G(\zeta,x)=\begin{cases}\varepsilon x^{-1}\sin(1-\zeta)\,x\,, & 0\le\zeta\le 1\,,\\ \varepsilon x^{-1}\sin x\cos\left(\dfrac{\zeta x}{\sqrt{k^*}}\right)-x^{-1}\cos x\sin\left(\dfrac{\zeta x}{\sqrt{k^*}}\right), & -\infty<\zeta\le 0\,,\end{cases} \tag{25}$$

$$Q(\zeta,x)=\begin{cases}\cos(1-\zeta)\,x\,, & 0\le\zeta\le 1\,,\\ -\cos x\cos\left(\dfrac{\zeta x}{\sqrt{k^*}}\right)-\varepsilon\sin x\sin\left(\dfrac{\zeta x}{\sqrt{k^*}}\right), & -\infty<\zeta\le 0\,.\end{cases} \tag{26}$$

Taking into account functions (23)–(26), we find by formulas (13) and (19) the temperature and intensities of the heat flows in the strip and the foundation under the condition of maintaining on the upper surface of the strip the initial temperature. Formulas (23)–(26) were obtained by us earlier from the solution of the thermal problem of friction with account for the heat transfer through the strip-foundation contact surface [8].

Asymptotic Solutions. At large values of the parameter of the integral Laplace transform (9) the temperature transforms (10)–(12) and those of the corresponding heat flow intensities take on the form

$$\overline{T}^{*}(\zeta, p) \cong \begin{cases} \dfrac{1}{(1+\varepsilon)\, p\sqrt{p}} \exp(-\zeta\sqrt{p}), & 0 \le \zeta \le 1, \\ \dfrac{1}{(1+\varepsilon)\, p\sqrt{p}} \exp\left(\zeta\sqrt{\dfrac{p}{k^{*}}}\right), & -\infty < \zeta \le 0, \end{cases} \tag{27}$$

$$\overline{q}^{*}(\zeta, p) \cong \begin{cases} \dfrac{1}{(1+\varepsilon)\, p} \exp(-\zeta\sqrt{p}), & 0 \le \zeta \le 1, \\ \dfrac{1}{(1+\varepsilon)\, p\sqrt{p}} \exp\left(\zeta\sqrt{\dfrac{p}{k^{*}}}\right), & -\infty < \zeta \le 0. \end{cases} \tag{28}$$

Using the relations [10]

$$L^{-1}\left[\frac{1}{p}\exp(-|\zeta|\sqrt{p});\tau\right] = \mathrm{erfc}\left(\frac{|\zeta|}{2\sqrt{\tau}}\right), \quad L^{-1}\left[\frac{1}{p\sqrt{p}}\exp(-|\zeta|\sqrt{p});\tau\right] = 2\sqrt{\tau}\,\mathrm{ierfc}\left(\frac{|\zeta|}{2\sqrt{\tau}}\right),$$

from relations (27) and (28) we find the asymptotic, for small dimensionless times τ, expressions for the dimensionless temperature and intensities of the heat flows

$$T^{*}(\zeta, \tau) \cong \begin{cases} \dfrac{2\sqrt{\tau}}{(1+\varepsilon)}\,\mathrm{ierfc}\left(\dfrac{\zeta}{2\sqrt{\tau}}\right), & 0 \le \zeta \le 1, \\ \dfrac{2\sqrt{\tau}}{(1+\varepsilon)}\,\mathrm{ierfc}\left(-\dfrac{\zeta}{2\sqrt{k^{*}\tau}}\right), & -\infty < \zeta \le 0, \end{cases} \tag{29}$$

$$q^{*}(\zeta, \tau) \cong \begin{cases} \dfrac{1}{(1+\varepsilon)}\,\mathrm{erfc}\left(\dfrac{\zeta}{2\sqrt{\tau}}\right), & 0 \le \zeta \le 1, \\ \dfrac{\varepsilon}{(1+\varepsilon)}\,\mathrm{erfc}\left(-\dfrac{\zeta}{2\sqrt{\tau}}\right), & -\infty < \zeta \le 0. \end{cases} \tag{30}$$

At $\xi = 0$, from solutions (29) and (30) we obtain formulas for the temperature and intensities of the heat flows on the contact surface

$$T^{*}(0+, \tau) = T^{*}(0-, \tau) \cong \frac{2}{(1+\varepsilon)}\sqrt{\frac{\tau}{\pi}}, \quad q^{*}(0+, \tau) = \frac{1}{1+\varepsilon}, \quad q^{*}(0-, \tau) = \frac{\varepsilon}{1+\varepsilon},$$

pointing to the fulfillment of the boundary conditions (3) and (4).

Note that formulas (29) and (30) completely coincide with the solution of the corresponding thermal problem of friction for two half-spaces [3, 13].

The asymptotic expressions of the temperature transforms (10)–(12) and of the corresponding heat flow intensities at small values of the parameter p of the integral Laplace transform (9) have the form

$$\overline{T}^{*}(\zeta, p) \cong \begin{cases} \dfrac{1 + \mathrm{Bi_s}(1-\zeta)}{\varepsilon(1+\mathrm{Bi_s})\, p\, (\alpha + \sqrt{p})}, & 0 \le \zeta \le 1, \\ \dfrac{1}{\varepsilon p\, (\alpha + \sqrt{p})}\left(1 + \dfrac{\zeta}{\sqrt{k^{*}}}\sqrt{p}\right), & -\infty < \zeta \le 0, \end{cases} \tag{31}$$

$$\bar{q}^{*}(\zeta, p) \cong \begin{cases} \dfrac{(1-\zeta)\,p + \mathrm{Bi_s}}{\varepsilon\,(1+\mathrm{Bi_s})\,p\,(\alpha+\sqrt{p})}, & 0 \le \zeta \le 1, \\ \dfrac{1}{\sqrt{p}\,(\alpha+\sqrt{p})}\left(1 + \dfrac{\zeta}{\sqrt{k^*}}\sqrt{p}\right), & -\infty < \zeta \le 0, \end{cases} \tag{32}$$

where

$$\alpha = \frac{\mathrm{Bi_s}}{\varepsilon\,(1+\mathrm{Bi_s})}. \tag{33}$$

Using the transition formulas [10]

$$\begin{aligned} L^{-1}\left[\frac{1}{\alpha+\sqrt{p}};\tau\right] &= \frac{1}{\sqrt{\pi\tau}} - \alpha\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau}), \\ L^{-1}\left[\frac{1}{\sqrt{p}\,(\alpha+\sqrt{p})};\tau\right] &= \exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau}), \\ L^{-1}\left[\frac{1}{p\,(\alpha+\sqrt{p})};\tau\right] &= 1 - \exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau}) \end{aligned} \tag{34}$$

from relations (31) and (32) we obtain asymptotic expressions for the dimensionless temperature and intensity of the heat flows in the strip and in the foundation for large dimensionless times τ:

$$T^{*}(\zeta,\tau) \cong \begin{cases} \dfrac{[1+\mathrm{Bi_s}(1-\zeta)]}{\mathrm{Bi_s}}\,[1-\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})], & 0 \le \zeta \le 1, \\ \dfrac{1}{\alpha\varepsilon}\left[1 - \left(1 - \alpha\dfrac{\zeta}{\sqrt{k^*}}\right)\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})\right], & -\infty < \zeta \le 0, \end{cases} \tag{35}$$

$$q^{*}(\zeta,\tau) \cong \begin{cases} \dfrac{\alpha(1-\zeta)}{\mathrm{Bi_s}}\left[\dfrac{1}{\sqrt{\pi\tau}} - \alpha\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})\right] + 1 - \exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau}), & 0 \le \zeta \le 1, \\ \exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau}) + \dfrac{\zeta}{\sqrt{k^*}}\left[\dfrac{1}{\sqrt{\pi\tau}} - \alpha\exp(\alpha^2\tau)\,\mathrm{erfc}\,(\alpha\sqrt{\tau})\right], & -\infty < \zeta \le 0. \end{cases} \tag{36}$$

Restricting ourselves to the first term in the expansion [12]

$$\sqrt{\pi}\,x\exp(x^2)\,\mathrm{erfc}\,(x) \approx 1 + \sum_{m=1}^{\infty}(-1)^m\frac{(2m-1)!!}{(2x^2)^m}, \tag{37}$$

we write formulas (35) and (36) in view of the designation of (33) in the form

$$T^{*}(\zeta,\tau) \cong \begin{cases} \dfrac{[1+\mathrm{Bi_s}(1-\zeta)]}{\mathrm{Bi_s}}\left[1 - \dfrac{\varepsilon\,(1+\mathrm{Bi_s})}{\mathrm{Bi_s}\sqrt{\pi\tau}}\right], & 0 \le \zeta \le 1, \\ \dfrac{1+\mathrm{Bi_s}}{\mathrm{Bi_s}}\left\{1 - \left[\dfrac{\varepsilon\,(1+\mathrm{Bi_s})}{\mathrm{Bi_s}} - \dfrac{\zeta}{\sqrt{k^*}}\right]\dfrac{1}{\sqrt{\pi\tau}}\right\}, & -\infty < \zeta \le 0, \end{cases} \tag{38}$$

$$q^*(\zeta,\tau) \cong \begin{cases} 1 - \dfrac{\varepsilon\,(1+\mathrm{Bi_s})}{\mathrm{Bi_s}\sqrt{\pi\tau}}, & 0 \le \zeta \le 1, \\[2ex] \dfrac{\varepsilon\,(1+\mathrm{Bi_s})}{\mathrm{Bi_s}\sqrt{\pi\tau}}, & -\infty < \zeta \le 0. \end{cases} \tag{39}$$

Assuming in solutions (38) and (39) first $\zeta = 0$ and then $\zeta = 1$, we find that the boundary conditions (3) and (4) on the contact surface and the condition of convective heat exchange on the outer surface of the layer (5) are fulfilled.

Passing in formulas (38) and (39) to the limit $\mathrm{Bi_s} \to \infty$, we obtain the dimensionless temperature and intensities of the heat flows while maintaining in the process of sliding on the outer surface of the strip the initial (zero) temperature

$$T^*(\zeta,\tau) \cong \begin{cases} (1-\zeta)\left[1 - \dfrac{\varepsilon}{\sqrt{\pi\tau}}\right], & 0 \le \zeta \le 1, \\[2ex] 1 - \left[\varepsilon - \dfrac{\zeta}{\sqrt{k^*}}\right]\dfrac{1}{\sqrt{\pi\tau}}, & -\infty < \zeta \le 0, \end{cases} \qquad q^*(\zeta,\tau) \cong \begin{cases} 1 - \dfrac{\varepsilon}{\sqrt{\pi\tau}}, & 0 \le \zeta \le 1, \\[2ex] \dfrac{\varepsilon}{\sqrt{\pi\tau}}, & -\infty < \zeta \le 0. \end{cases} \tag{40}$$

Formulas (40) were obtained from the solution of the thermal friction problem with account for the heat transfer through the strip-foundation contact surface [8].

To obtain the corresponding solutions in the case of heat insulation of the outer surface of the strip, let us pass to the limit $\mathrm{Bi_s} \to 0$ in formulas (10)–(12). As a result, we have

$$\overline{T}^*(\zeta,p) \cong \begin{cases} \dfrac{1}{p\sqrt{p}\,(\varepsilon+\sqrt{p})}, & 0 \le \zeta \le 1, \\[2ex] \dfrac{1}{p\sqrt{p}\,(\varepsilon+\sqrt{p})}\left(1 + \zeta\sqrt{\dfrac{p}{k^*}}\right), & -\infty < \zeta \le 0. \end{cases} \tag{41}$$

Using the second formula from the transition formulas (34) and additionally [10]

$$L^{-1}\left[\frac{\varepsilon^2}{p\sqrt{p}\,(\varepsilon+\sqrt{p})};\tau\right] = 2\varepsilon\sqrt{\frac{\tau}{\pi}} - 1 + \exp(\varepsilon^2\tau)\,\mathrm{erfc}\,(\varepsilon\sqrt{\tau})$$

from relations (41) we find

$$T^*(\zeta,\tau) \cong \begin{cases} \dfrac{2}{\varepsilon}\sqrt{\dfrac{\tau}{\pi}} - \dfrac{1}{\varepsilon^2}\left[1 - \exp(\varepsilon^2\tau)\,\mathrm{erfc}\,(\varepsilon\sqrt{\tau})\right], & 0 \le \zeta \le 1, \\[2ex] \dfrac{2}{\varepsilon}\sqrt{\dfrac{\tau}{\pi}} - \dfrac{1}{\varepsilon}\left(\dfrac{1}{\varepsilon} - \dfrac{\zeta}{\sqrt{k^*}}\right)\left[1 - \exp(\varepsilon^2\tau)\,\mathrm{erfc}\,(\varepsilon\sqrt{\tau})\right], & -\infty < \zeta \le 0. \end{cases} \tag{42}$$

In view of expansion (37), let us give the dimensionless temperature (42) in the final form:

$$T^*(\zeta,\tau) \cong \begin{cases} \dfrac{2}{\varepsilon}\sqrt{\dfrac{\tau}{\pi}} - \dfrac{1}{\varepsilon^2}\left(1 - \dfrac{1}{\varepsilon\sqrt{\pi\tau}}\right), & 0 \le \zeta \le 1, \\[2ex] \dfrac{2}{\varepsilon}\sqrt{\dfrac{\tau}{\pi}} - \dfrac{1}{\varepsilon}\left(\dfrac{1}{\varepsilon} - \dfrac{\zeta}{\sqrt{k^*}}\right)\left(1 - \dfrac{1}{\varepsilon\sqrt{\pi\tau}}\right), & -\infty < \zeta \le 0. \end{cases}$$

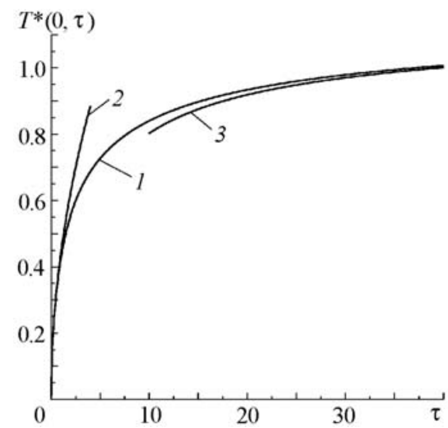


Fig. 2. Evolution of the dimensionless temperature T^* on the friction surface $\zeta = 0$ of the strip and the foundation for the Biot number $Bi_s = 5$: 1) exact solution (13)–(16); 2, 3) asymptotic solutions for small (29) and large (38) values of the Fourier number τ, respectively.

Braking Temperature of the Lining-Disc Tribosystem. Suppose that the sliding velocity of the strip (lining) on the foundation (disc) surface is not constant but decreases from the maximum value of V_0 at the initial instant of time $t = 0$ to zero at the stop moment $t = t_b$ (constant deceleration breaking)

$$V(t) = V_0\left(1 - \frac{t}{t_b}\right), \quad 0 \le t \le t_b . \tag{43}$$

In this case, the formulation of the thermal friction problem has the form (1)–(7), with the sum of dimensionless intensities of normal heat flows from the friction surface into the strip and the foundation on the right side of the boundary conditions (4) being equal not to unity, but to the function

$$q^*(\tau) = 1 - \frac{\tau}{\tau_b}, \quad \tau_b = \frac{k_s t_b}{d^2}, \quad 0 \le \tau \le \tau_b . \tag{44}$$

We find the dimensionless braking temperature T_b^* in the lining and in the disc under braking with constant deceleration (43) with the aid of the Duhamel theorem [14]

$$T_b^*(\zeta, \tau) = \int_0^\tau q^*(s) \frac{\partial}{\partial s} T^*(\zeta, \tau - s)\, ds, \quad -\infty < \zeta \le 1, \quad 0 \le \tau \le \tau_b . \tag{45}$$

Substituting into the right side of inequality (44) the dimensionless intensity of the heat flow $q^*(\tau)$ (44) and the dimensionless temperature $T^*(\zeta, \tau)$ (13), (14), upon integration with respect to s we get

$$T_b^*(\zeta, \tau) = \frac{2}{\pi}\int_0^\infty F(x)\, G(\zeta, x)\, Q(\tau, x)\, dx, \quad -\infty < \zeta \le 1, \quad 0 \le \tau \le \tau_b , \tag{46}$$

where

$$Q(\tau, x) = 1 - \frac{\tau}{\tau_b} - \exp(-x^2\tau) + \frac{1 - \exp(-x^2\tau)}{x^2\tau_b}, \quad Q(\tau, 0) = 0 , \tag{47}$$

and the functions $F(x)$ and $G(\zeta, x)$ have the form (15) and (16), respectively.

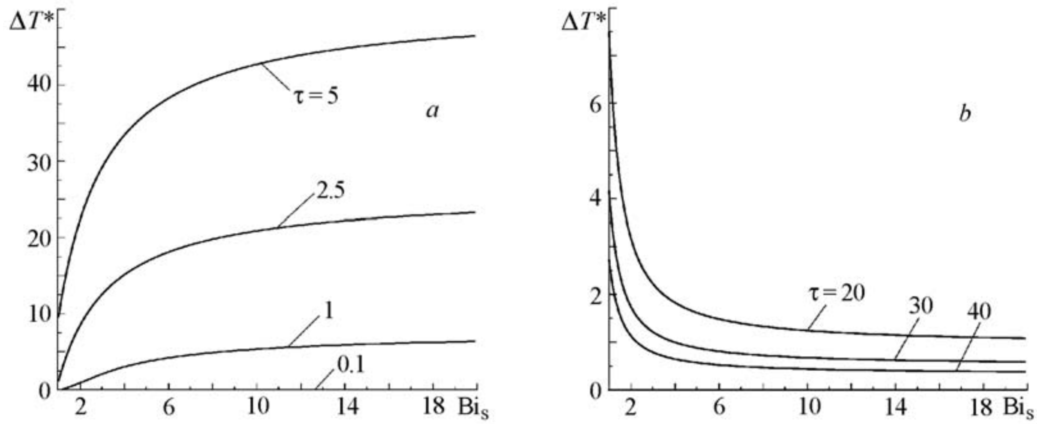
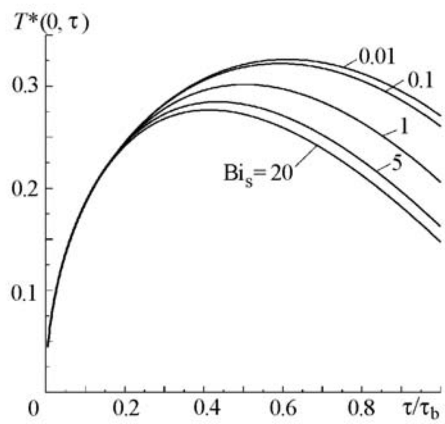


Fig. 3. Relative error ΔT^* (48) at small (a) and large (b) values of Fourier numbers τ versus the Biot number Bi_s.



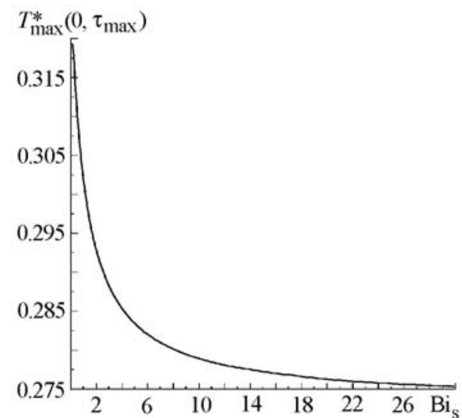


Fig. 4. Evolution of the dimensionless temperature T_b^* on the contact surface $\zeta = 0$ for several values of the Biot number Bi_s.

Fig. 5. Dimensionless maximum temperature $T_{b.max}^*$ at a dimensionless braking time $\tau_b = 2.08$ versus the Biot number Bi_s.

Numerical Analysis. We have performed calculations of the dimensionless temperatures T^* and T_b^* for an FMK-11 cermet strip ($K_s = 34.31$ W/(m·K), $k_s = 15.2 \cdot 10^{-6}$ m^2/sec) of thickness $d = 5$ mm and a ChKhMK cast iron foundation ($K_f = 51$ W/(m·K), $k_f = 14 \cdot 10^{-6}$ m^2/sec [15].

The evolution of the dimensionless temperature T^* on the contact surface $\zeta = 0$ for a Biot number $Bi_s = 5$ is shown in Fig. 2. Calculations have been made with the use of the formulas of the exact solution of the problem (13)–(16), as well as of the asymptotic solutions for small (29), (30) and large (38), (39) Fourier numbers of τ.

Figure 3 shows the dependences of the quantity

$$\Delta T^* = \left| [T^*(0, \tau) - \tilde{T}^*(0, \tau)] / T^*(0, \tau) \right| \cdot 100\% , \tag{48}$$

where $T^*(0, \tau)$ is the contact surface temperature found by the formulas of the exact solution (13)–(15), (17) and $\tilde{T}^*(0, \tau)$ is the temperature calculated with the aid of asymptotic solutions for small (29) or large (38) values of the dimensionless time τ depending on the Biot number Bi_s. An increase in the heat exchange on the outer surface of the strip for values $0 < \tau \le 5$ leads to an increase in ΔT^* (Fig. 3a), and at $\tau \ge 20$, to a decrease in this characteristic (Fig. 3b). Consequently, the application of the asymptotic solution for small values of Fourier numbers (29) to the temperature calculation on the friction surface is preferable for small ($0 \le Bi_s \le 2$) values of the Biot number, and the use of asymptotic solutions for large Fourier numbers (38) is preferable for $Bi_s \ge 10$.

Calculations of the dimensionless braking temperature T_b^* (46) have been performed for a Fourier number τ_b = 2.08 corresponding to the braking period t_b = 3.42 sec. The integral in formula (46) was found by the QAGI procedure from the QUADPACK numerical integration package.

At the start of braking, the temperature on the contact surface $\zeta = 0$ sharply increases, reaches its maximum value $T^*_{b.max}$ at the instant of time τ_{max}, and then begins to decrease to the minimum value at the stop moment τ_b (Fig. 4). The heat exchange with the environment on the outer surface of the strip has no practical effect on the temperature in the initial period of braking when the temperature increases with time. This influence is the strongest in the period of contact surface cooling after the temperature reaches the maximum value.

The highest temperature on the contact surface is attained in the case of heat insulation of the outer surface of the strip ($Bi_s \to 0$) (Fig. 5). As the heat transfer from the outer surface of the strip into the environment increases, the maximum temperature on the contact surface decreases. From the data presented in Fig. 5 it follows that for Biot number values $Bi_s \geq 20$, to determine the maximum temperature in the considered tribosystem, we can make use of the analytical solution of the problem at a given zero temperature on the outer surface of the strip [5].

Conclusions. We have obtained an analytical solution of the thermal friction problem for a plane-parallel strip-foundation tribosystem in which the convective heat exchange with the environment on the upper (free) surface of the strip is taken into account. Asymptotic formulas for calculating the temperatures and intensities of heat flows of the strip and the foundation at small and large values of the Fourier number have been given. The validity range of these formulas have been established. The influence of the Biot number on the evolution of the friction surface temperature in the process of braking with constant deceleration of the cermet strip (lining) and the cast iron foundation (disc) has been investigated.

NOTATION

Bi_s, Biot number; d, strip thickness; erf (x), Gauss error function; erf (x) = 1 − erf (x); ierfc (x) = $\pi^{-1/2}$ exp $(-x^2) - x$ erfc (x); f, friction coefficient; h_s, heat transfer coefficient; K, heat conductivity coefficient; k, thermal diffusivity; L, L^{-1}, direct and inverse Laplace transform; m, summation index in the series m = 1, 2, 3, ...; p, parameter of integral Fourier transform; p_0, external pressure; q, heat flow intensity; q_0, specific friction power; q^*, dimensionless intensity of the heat flow; s, integration variable; T, temperature; T_0, factor having temperature dimensionality; t, time; t_b, braking time; V, sliding velocity; V_0, initial sliding velocity; z, spatial variable; ε, thermal activity coefficient of the foundation with respect to the strip; ζ, dimensionless coordinate; τ, τ_b, Fourier numbers. Subscripts: b, braking; f, foundation; s, strip.

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